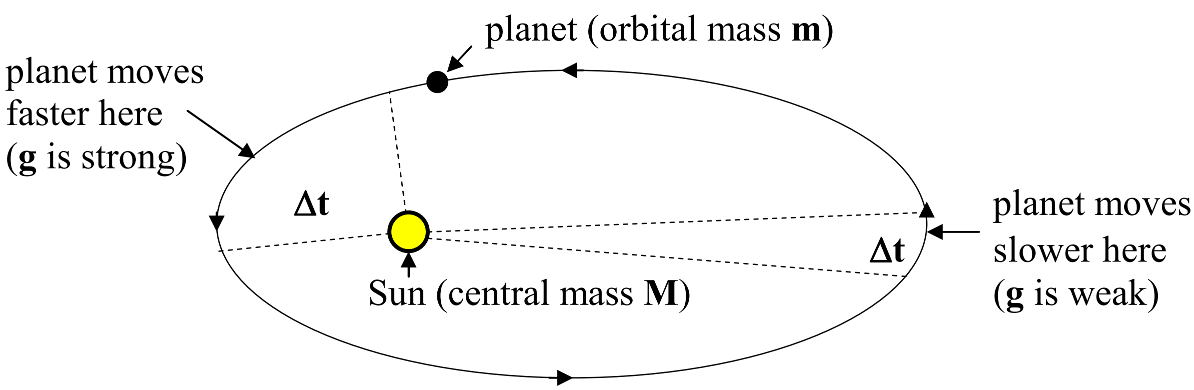
Kepler's Three Laws of Planetary Motion

In the 1700's, Johannes Kepler used Newton's work to establish the nature of a body's orbital path around a central mass. This is what he found:

1. Planetary orbits are *elliptical*, with the sun at one focus.
   - Note that many orbits are very nearly circular; for such cases, we can state that $\mathbf{F_c = F_g}$
2. A line joining the center of any planet with the sun, traces out equal areas in equal times.
   - This statement tells us that for very elliptical paths, a planet travels at greatest speed *closest* to the Sun (where '**g**' is greatest) and travels slowest at its *furthest* point from the Sun (where 'BgB' is weakest).
3. For a given central mass, if **R = the average orbital radius**, and **T** = **the orbital period**, then the ratio of $\mathbf{\frac{R^3}{T^2}}$ is a constant.



Newton's Synthesis

We can derive Kepler's Third Law from Newton's Second Law and find a way to get Kepler's Constant. Start by examining a satellite of mass '**m**' travelling in a stable orbit with period of revolution '**T**' around a central planetary mass '**M**'.

If we assume the elliptical orbit is very nearly circular, then once again $\mathbf{F_c = F_g}$.

- combine $\mathbf{F_g = \frac{GMm}{R^2}}$ and $\mathbf{F_c = m\frac{4\pi^2 R}{T^2}}$: $\mathbf{\frac{GMm}{R^2} = m\frac{4\pi^2 R}{T^2}}$
- cancel orbital mass '**m**' and rearrange to get $\mathbf{\frac{R^3}{T^2} = \frac{GM}{4\pi^2}}$
- **G**, **M**, and $\mathbf{4\pi^2}$ are all constant (remember, '**M**' is the central mass), so Kepler's Constant is therefore $\mathbf{k = \frac{GM}{4\pi^2}}$.

Example 7: A telecommunications satellite orbits the Earth once every 24 hours in what is called a <u>geosynchronous</u> orbit. What is the altitude of this satellite?

(see Gravitation Ex 7 for answer)

To use Kepler's Third Law, you must have **R** and **T** data for one satellite so that you can find the constant **k**, or you can use ratio solutions.

If $\frac{R^3}{T^2} = k$, then $R^3 = kT^2$

We now have $\left(\frac{R_B}{R_A}\right)^3 = \left(\frac{T_B}{T_A}\right)^2$ → use this to solve using ratios

Note: In order to simplify meaning, the period (year) and radius measurements of planets are often made in terms of Earth measurements. A planet's year may be given in terms of Earth days or years. A planet's distance from the Sun may be given in terms of Earth's distance from the Sun. This distance is called one **astronomical unit** (**AU**).

Example 8: Mercury's Year = 88 Earth days and its distance from the Sun is 0.37 AU. Find the orbital radius of Venus if its year = 224.7 Earth days.

(see Gravitation Ex 8 for answer)